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A STRUCTURED FLUIDS APPROACH TO CANOPY FLOW

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Abstract

The "classical" turbulence approach to the flow of air in a vegetative canopy is reviewed. Three separate "classical" theories are described. A comparison is made between experimentally measured canopy flows and a velocity function derived from the theory of structured continua. The agreement between theory and observations is good, but not unexpected, due to seven degrees of freedom in the fitted velocity profile. Proposals for future study for a more rigorous test of the theory are discussed.

I. Introduction

The purpose of this paper is to apply the theory of structured fluids to the turbulent flow of air in a vegetative canopy. The model applied here is the flow between two plates of fluid containing deformable structures. It is believed that this represents the first attempt to compare the results of experiment with the theory of structured fluids.

First, the classical approach to canopy flow is discussed. Then the theory of structured fluids is reviewed and the general solution to the parallel plate problem presented. This is followed by a comparison between experimental measurements and the theory of structured fluids. Finally, the results are summarized and discussed.

II. Classical Approach to Canopy Flow

The mean flow of air in vegetative canopy has been studied "classically" by Ordway, Ritter, Spence and Tan [1]. They consider the problem essentially the same as the classical sliding plate problem: "At the upper edge of the flow, say $z = z_h$ we see that the outside turbulent flow effectively drags the canopy flow, analogous to a moving wall with velocity U_h ... " [1].

The basic differential equation of canopy flow is derived in [1]. The principal result is:

$$\frac{\partial}{\partial z}\left(\epsilon\,(z) + \mu\right)\frac{\partial U}{\partial z} = -\frac{A(z)\,C_D^{'}\,\rho\,U^2}{2}, \qquad (1)$$

where

z = vertical coordinate,

μ = molecular viscosity coefficient (a constant),

$\epsilon(z)$ = eddy viscosity coefficient (a function of z),

$A(z)$ = total vertical plane area of the plants comprising the canopy per unit volume (a function of z),

$C_D^{'}$ = drag coefficient of the plants,

ρ = mass density of air,

U = mean horizontal wind velocity (a function of z).

The physical interpretation of this equation is that the divergence of the Reynold's stress is equal to the fluid drag.

Before equation (1) can be evaluated, $\epsilon(z)$, $A(z)$, and $C_D^{'}$ must be determined. For realistic values of these functions equation (1) cannot be solved analytically and it was necessary to resort to a numerical technique.

Cionco [2, 3] has treated this problem in a slightly different manner. Whereas [1] was concerned with the behavior of a "transfer coefficient" throughout the canopy [2, 3] concerned itself with the mixing length and its properties throughout the canopy. The canopy flow equation derived in [3] is:

$$0 = \left\{ \frac{C}{G} U^2 - 2 \frac{\partial U}{\partial X} \left[\frac{\partial U}{\partial X} \frac{\partial \ln \ell_c}{\partial X} + \frac{\partial^2 U}{\partial X^2} \right] \right\}, \qquad (2)$$

where

ℓ_c = mixing length of the canopy (a function of z),

$X = z/h$.

C and G are two parameters introduced by Cionco, their definitions are quite lengthy and will not be presented here.

Equation (2) may be interpreted physically in the same manner as equation (1). It must also be evaluated numerically. The procedure is described in [3].

A purely empirical treatment of the problem was given by Uchijima and Wright [4]. In this approach the authors postulated a velocity function:

$$U = U_h + A_o \ln \frac{z}{h} - B(z), \qquad 0 \le z \le h \qquad (3)$$

where

A_o = a proportionality constant (a function of U_h),

$B(z)$ = the effect of the plant community on the wind (a function of z).

The authors took measurements of U and adjusted B(z) so that the equation would be satisfied. In effect B(z) represents the difference between the measured values of U and the first two terms on the right side of equation (3). It would appear that this procedure allows equation (3) to agree with any experimental data.

III. Structured Fluid Approach

Theory of structured continua

Basically this approach treats the air and plants as a structured continuum. The premise introduced here is that the bulk fluid velocity of the structured fluids approach may be identified with the mean velocity of the classical turbulence approach.

A continuum may be described as a medium in which the field quantities (displacements, stresses, velocities, etc.) are piecewise-continuous functions of the coordinates of the material points and time.

A structured continuum is a continuum that contains certain mathematical abstractions called "structures". At present there seems to be a number of possible physical interpretations for these structures, see [5, 6, 7] for example. However, none of these have been formalized. Perhaps some formal physical interpretation will evolve in the near future. It is clear as pointed out by Truesdell and Toupin [8] that with or without such an interpretation the mathematics of the theory retain their validity.

The axioms upon which the theory is based are:

(a) Conservation of Mass,

(b) Balance of Momentum,

(c) Balance of Moment of Momentum,

(d) Conservation of Energy,

(e) Principle of Entropy.

The discussion of structured fluids presented here has been necessarily brief. For the details and foundations the interested reader is referred to [5, 6, 7, 8, 9, 10, 11, 12].

Constitutive Equations and General Theory

When forces are applied to any material, axioms (a) through (e) must hold. However, when the same system of forces is applied to different materials the materials respond differently. In other words, the same system of forces applied to different materials give rise to different strains or rates of strain. The relation between stress and strain is referred to as the constitutive equations.

The constitutive equations for a structured fluid have been developed in [11], and the parallel plate problem has been solved for a fluid containing rigid structures [12] and for a fluid containing non-rigid structures [13]. The formulation leads to 3 coupled ordinary differential equations:

$$(a+b)\,D'' + 2(B_2 - E_2)\,D = 0, \tag{4}$$

$$(a-b)\,V'' - E_3 U' + 2E_3 V = 0, \tag{5}$$

$$(A_2 - E_3)\,U'' + 2E_2 D' + 2E_3 V' = P, \tag{6}$$

where

U = mean velocity of the fluid,

V = rate of rotation or vorticity of the structures,

D = deformation of the structures,

P = the pressure gradient,

a, b, B_2, E_2, E_3, A_2 are viscosity coefficients.

Equations (4) and (5) are statements of axiom (c) and equation (6) is a statement of axiom (b). Equation (4) is a result of considering deformable structures.

It should be pointed out that a and b have dimension L^4T^{-1} while B_2, E_2, E_3 and A_2 have dimension L^2T^{-1}. Kirwan [13] discusses the constraints on the viscosity coefficients due to the second law of thermodynamics.

The solutions to the above set of equations with z normalized were derived for the following boundary conditions in [13]

(a) $$U(z = 0) = U(z = 1) = 0$$

$$D(z = 0) = -D(z = 1) = D_0$$

$$V(z = 0) = -V(z = 1) = V_0$$

(b) $$P = 0$$

$$U(z = 0) = 0$$

$$U(z = 1) = U_1 > 0$$

$$D(z = 0) = D(z = 1) = D_0$$

$$V(z = 0) = V(z = 1) = V_0$$

The solution to this system of equations with general boundary conditions for the lower plate fixed and the upper one moving* is

$$U = \frac{P}{2A_2}(Z^2 - 1) + X_z + \frac{Y_2(D_0 - D_1)}{\alpha_5} r(Lz)$$

$$+ \frac{2E_3M}{\alpha_5} r(Kz) - \frac{2^{Y}2}{\alpha_5}(D_0 + D_1) R(Lz) - \frac{2E_3N}{\alpha_5} R(Kz), \qquad (7)$$

---

*The vertical coordinate, z, has been normalized for convenience.

$$V = Mf(Kz) + 2Y_1 D + \frac{P(2z - 1)}{4A_2} + Nh(Kz) + \frac{X}{2}, \tag{8}$$

$$D = (D_0 - D_1)\, f(Lz) + (D_0 + D_1)\, h(Lz). \tag{9}$$

Here:

$$U_{z=0} = U_0 = 0$$

$$U_{z=1} = U_1$$

$$V_{z=0} = V_0$$

$$V_{z=1} = V_1$$

$$D_{z=0} = D_0\,,$$

$$D_{z=1} = D_1\,,$$

$$L^2 = 2(B_2 - E_2)/(a + b)\,,$$

$$K^2 = -2E_3 A_2/(a + b)\,\alpha_5\,,$$

$$X = U_1 + \frac{4}{\alpha_5}\;\frac{Y_2}{2}\; S(L)\,(D_0 + D_1) + E_2\, NS(K)\,,$$

$$N = \frac{1}{\xi}\,\alpha_5\,(V_0 + V_1) - Y_1\,\alpha_5\,(D_0 + D_1) - \frac{U_1\alpha_5}{2} - 2Y_1\, S(L)\,(D_0 + D_1)\,,$$

$$M = (V_0 - V_1) - Y_1 (D_0 - D_1) + \frac{P}{4A_2} ,$$

$$g(\gamma z) = \frac{[\cosh \gamma(1-z) + \cosh (\gamma z)]}{\gamma \sinh \gamma} ,$$

$$S(\gamma z) = \frac{[\cosh (\gamma z) - \cosh \gamma(1-z)]}{\gamma \sinh \gamma} ,$$

$$r(\gamma z) = g(\gamma z) - g(\gamma) ,$$

$$R(\gamma z) = S(\gamma z) + S(\gamma) ,$$

$$f(\gamma z) = [\sinh \gamma(1-z) - \sinh (\gamma z)]/\sinh \gamma ,$$

$$h(\gamma z) = [\sinh (\gamma z) + \sinh \gamma(1-z)]/\sinh \gamma ,$$

$$Y_1 = E_2 K^2 / A_2 (L^2 - K^2) ,$$

$$Y_2 = 2(E_2 + E_3 Y_1) ,$$

$$\zeta = \alpha_5 + 2E_3 S(K) ,$$

$$\alpha_5 = A_2 \quad E_3 .$$

It is worthwhile noting that instead of reducing the problem to 3 coupled ordinary differential equations, one equation composed of higher order derivatives of U could have been obtained.

IV. Comparison Between Theory and Experiment

The first step in the comparison between experimental data and the theory of structured continua was to determine if the velocity function equation (7) is capable of producing velocity profiles similar to those measured*. In this paper only the velocity function $U(z)$ will be treated. A short cut in this comparison is found via the method of Least Squares. The trial function we would like to use is:

$$U = \beta_1(z^2 - 1) + B_2 z + \beta_3 r(Lz) + \beta_4 r(kz) + \beta_5 R(Lz) + \beta_6 R(kz) \tag{10}$$

The primary problem is to determine β_1, β_2, β_3, β_4, β_5, β_6, K and L from observations. Although for each profile there were only six independent measurements, thirty-six additional points along the profile were interpolated for use in the Least Squares procedure. The determination of K and L make this a nonlinear least square problem. While computational methods are in existence for this [14, 15] a quicker and easier procedure was used. Equation (10) was treated as a linear trial function with values for K and L specified. Likely values of K and L were used. Following this procedure a new trial function was introduced so as to facilitate programming:

---

*The data for this comparison was supplied by Lemon and Allen of the New York State College of Agriculture, Cornell University.

$$U = \beta_1' + \beta_2' z + \beta_3' z^2 + \beta_4' e^{-Lz} + \beta_5' e^{Lz} + \beta_6' e^{-Kz} + \beta_7' e^{Kz} \qquad (10a)$$

It should be noted that a seventh degree of freedom has been added. This is because we are no longer constraining $U(z = 0)$ to be 0 due to a lack of data in the lower portions of the canopy. Tests indicate that this constraint has little or no effect on the results. The results of these computations are illustrated graphically in Figures 1 through 9. The solid lines are the measured profiles and the broken lines are the fitted profiles. In the cases where only a solid line is plotted the fitted curve was so close to the measured curve that no distinction could be made graphically. A total of approximately twenty thousand runs were made. The correlation coefficients in every case were between .990 and .999.

As can be seen the agreement between equation (7) and the data is quite good. However, it was found that just as good as fit could be obtained with a sixth degree polynomial. It was also found that the fit with $L = 0$ (rigid structures) and a fourth order polynomial were nearly as good.

JAPANESE LARCH

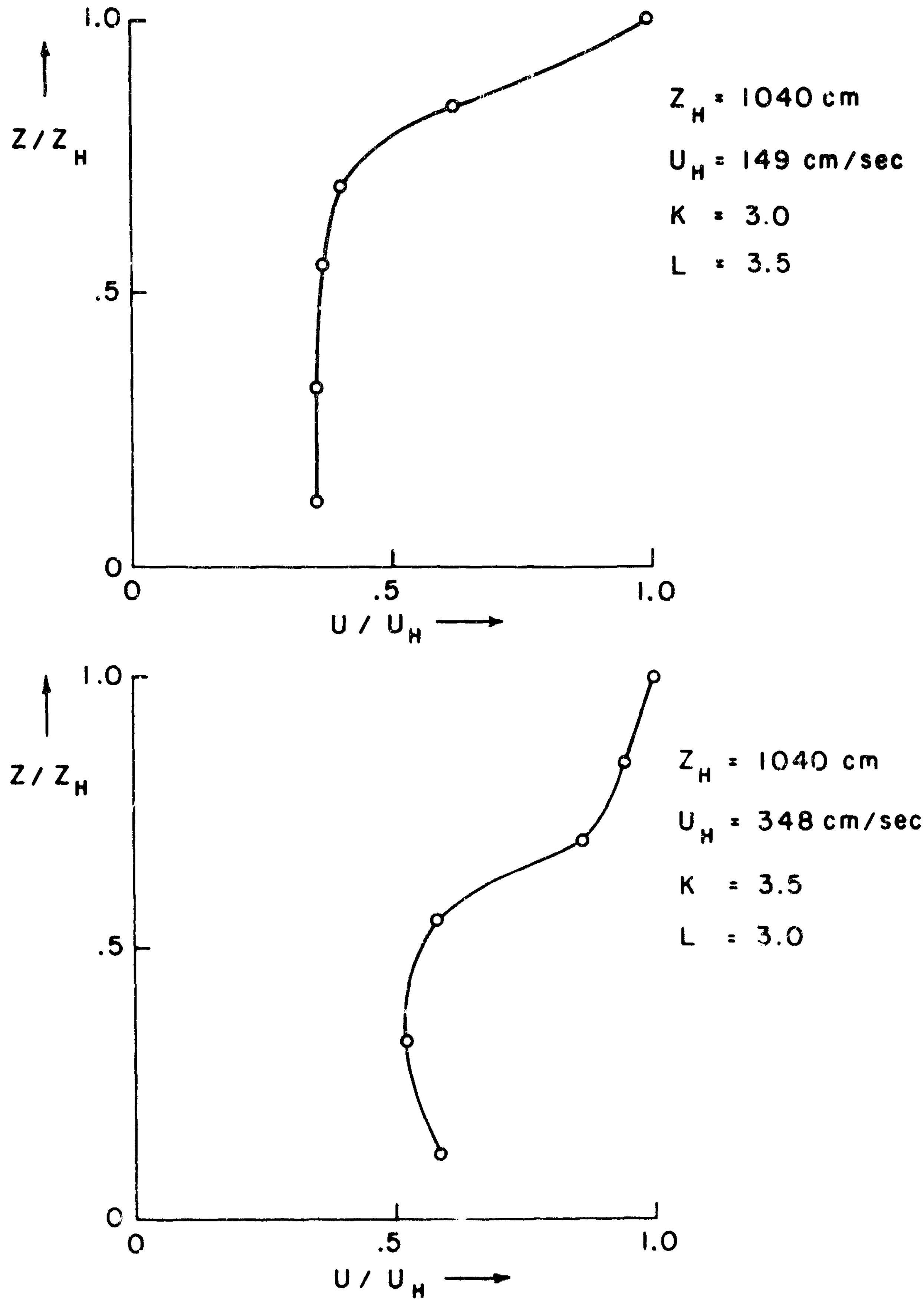
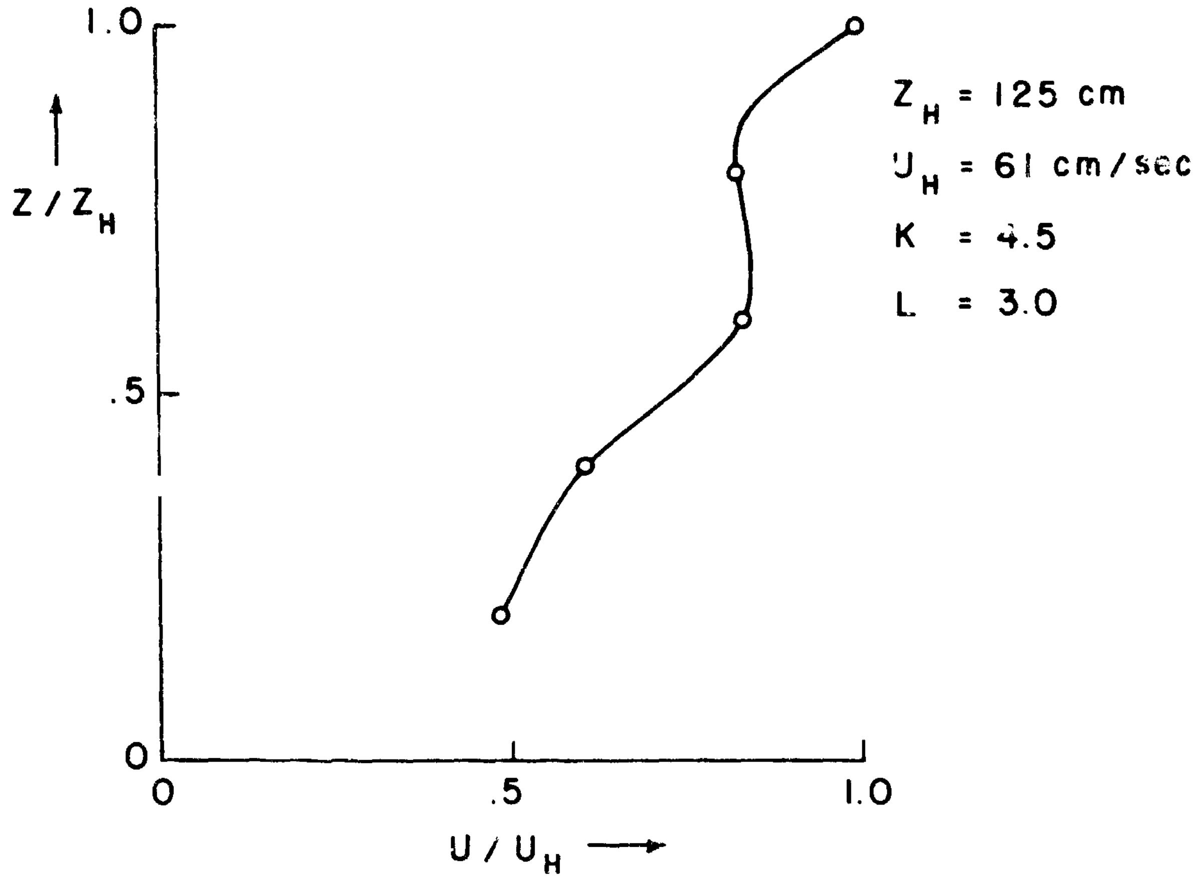


Figure 1 and 2. Mean horizontal normalized velocity profiles in the Japanese Larch canopy.

OATS



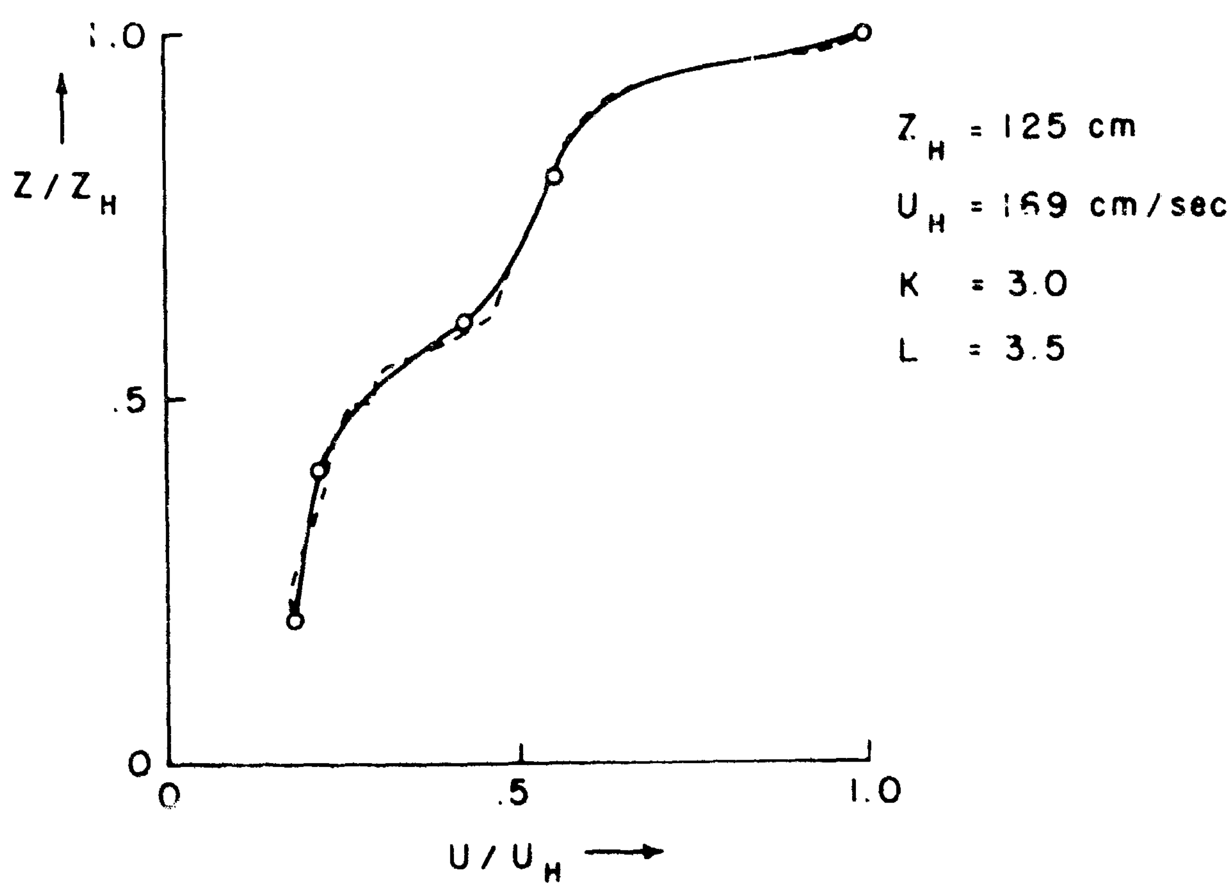


Figure 3 and 4. Mean horizontal normalized velocity profiles in the Oats canopy.

SOY BEANS

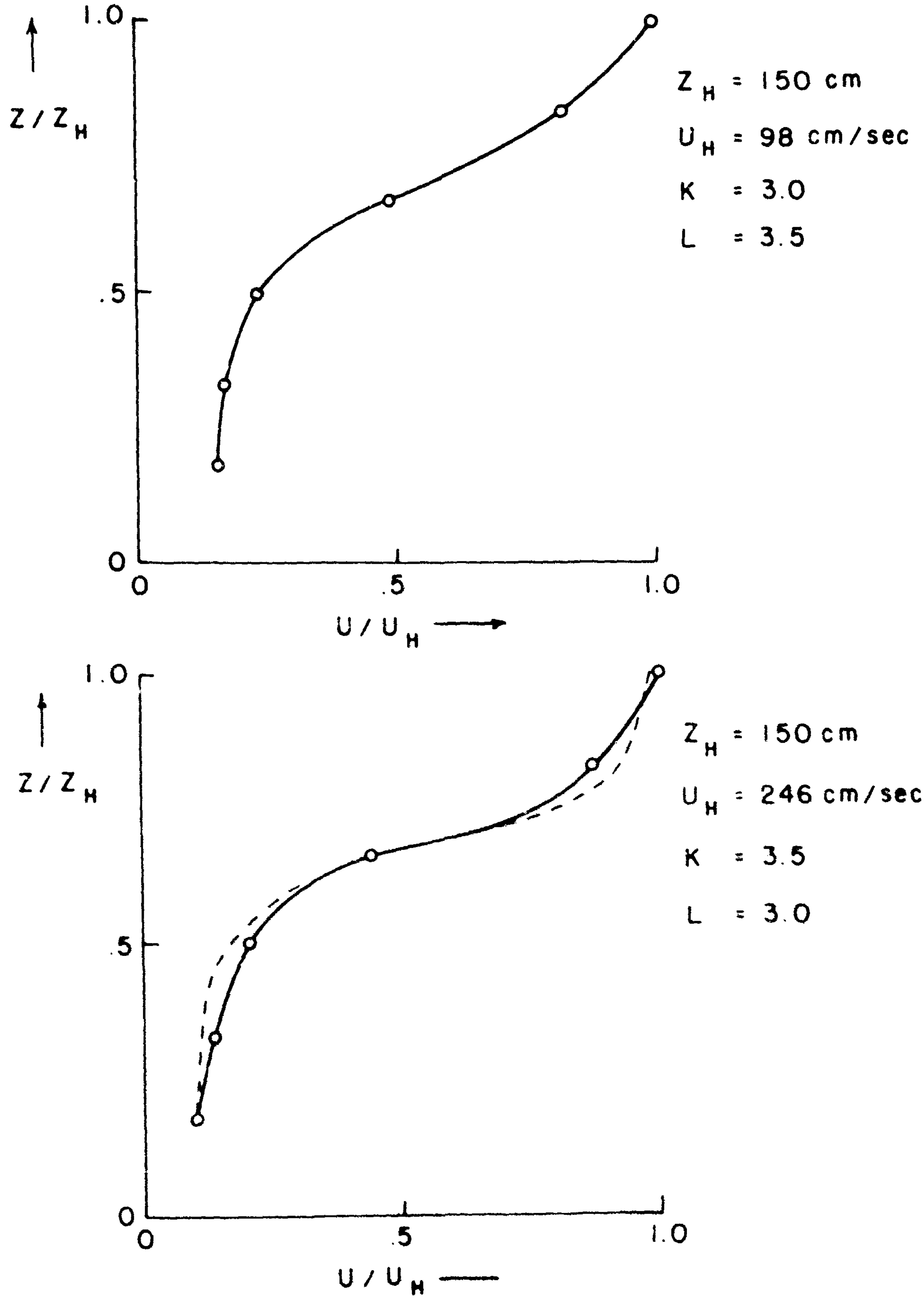
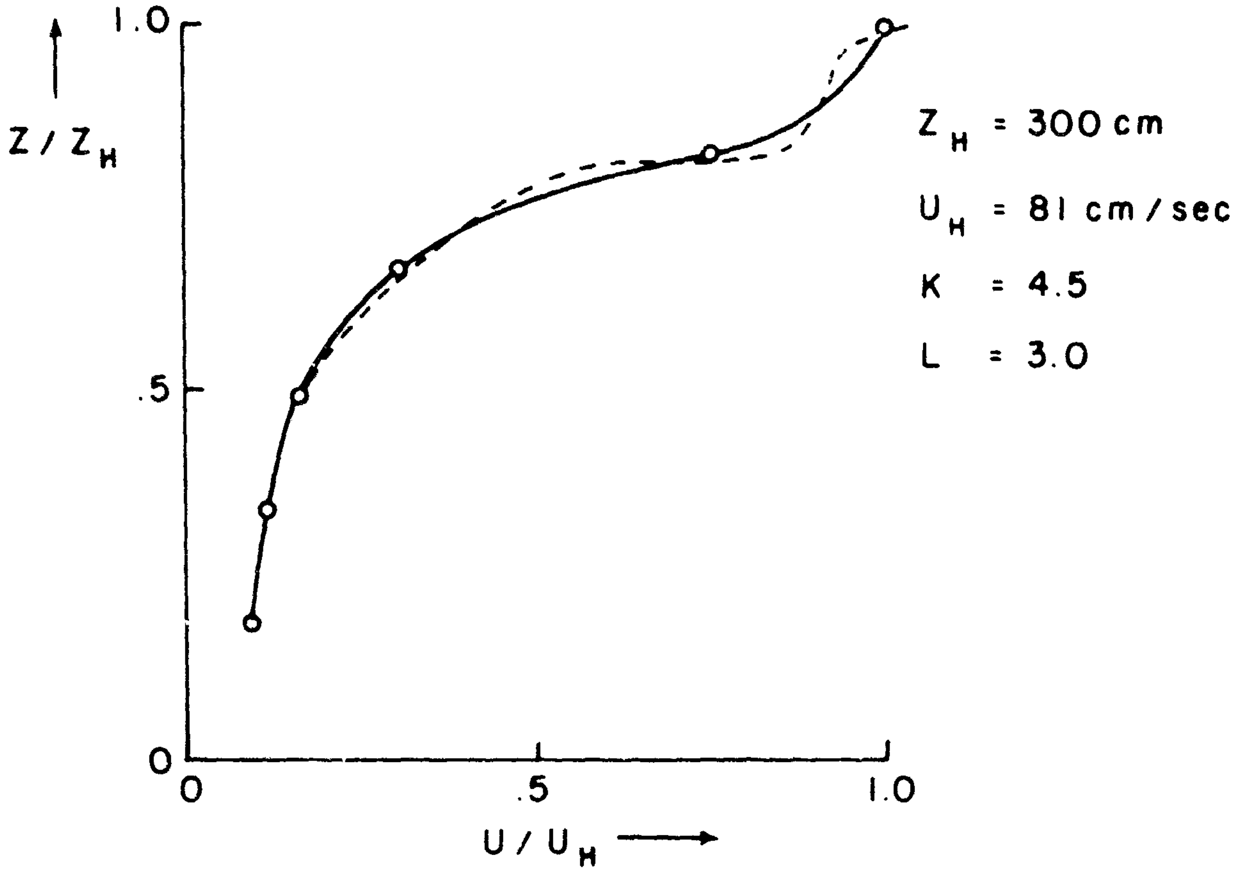


Figure 5 and 6. Mean horizontal normalized velocity profiles in the Soy Beans canopy.

CORN



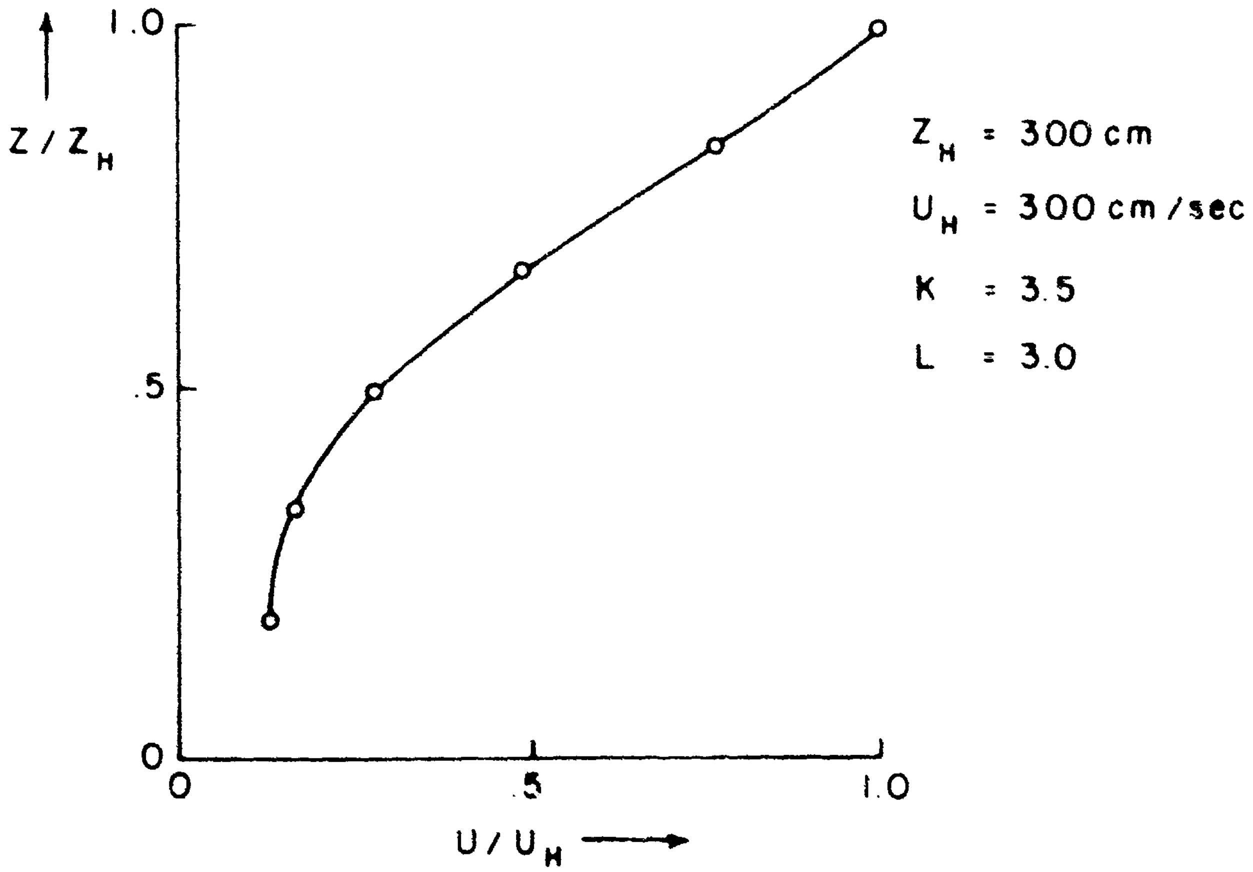


Figure 7 and 8. Mean horizontal normalized velocity profiles in the corn canopy.

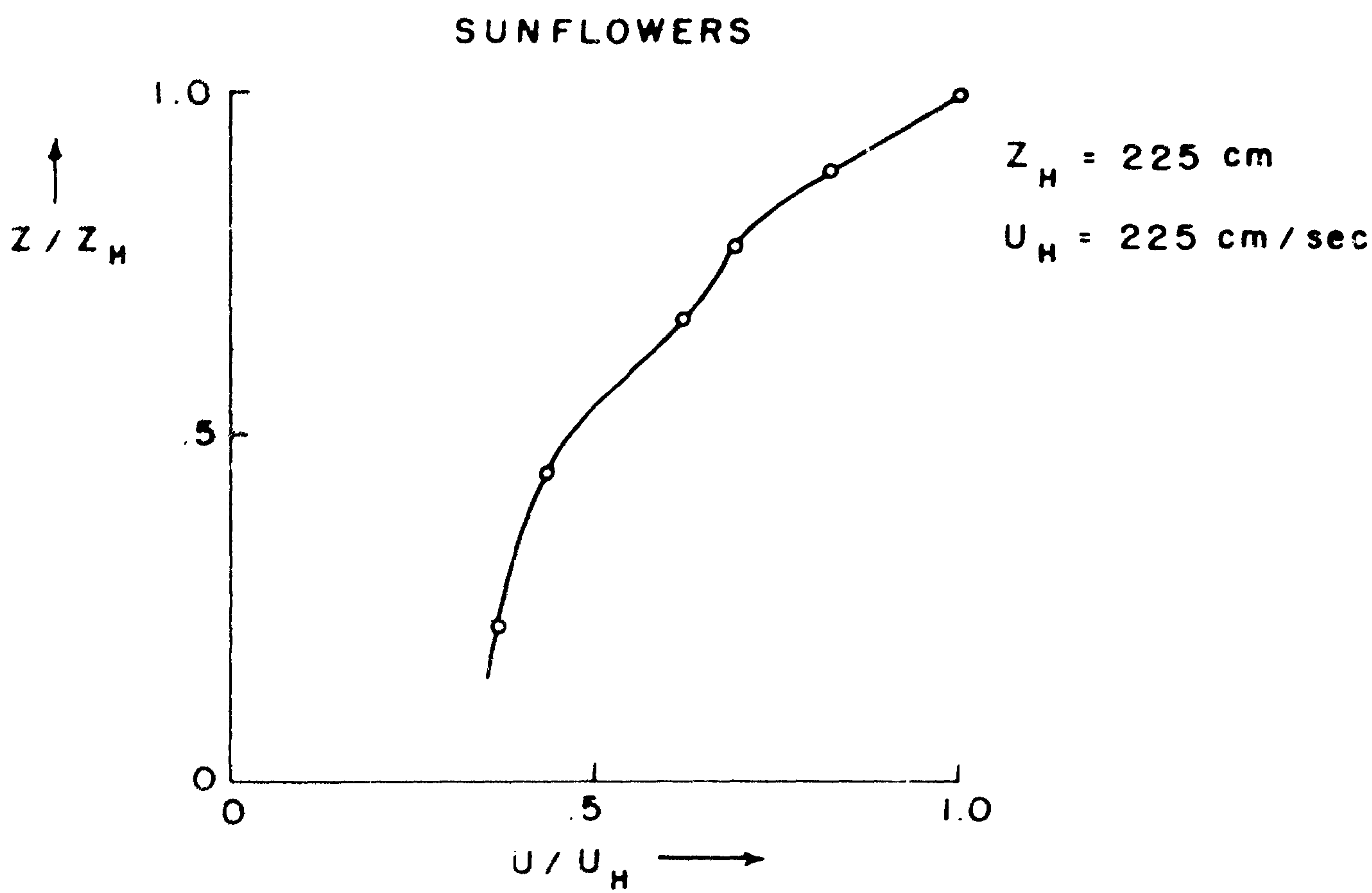


Figure 9. Mean horizontal normalized velocity profiles in the Sunflowers canopy.

The fact that an arbitrary trial function with the same number of degrees of freedom as the velocity function derived from the theory of structured continua can fit the data so well is to be expected since the degrees of freedom of the trial function is slightly greater than the number of independent data points. The advantage of using the velocity function derived here is that it is derived from a physical theory.

V. Discussion of Results

As can be seen from Figures 1 through 9, the velocity function derived via the theory of structured continua can quite accurately reproduce experimentally measured mean velocity profiles in various vegetative canopies. However, as mentioned previously, the fact that the trial function used in the least squares routine has seven degrees of freedom pretty much assures a good fit to the data. It is worth noting that neither the log wind nor the parabolic trial function could reproduce the experimentally measured profiles.

To summarize there are three methods of reproducing experimentally measured "Canopy Flows":

(a) "Classical Approach",

(b) Theory of Structured Continua,

(c) Arbitrary trial function with enough degrees of freedom.

Of the three, the last seems least attractive because it is completely arbitrary and can be related to no physical concepts. The first one suffers from what might be called "creeping empiricism". Such features as a height dependent eddy viscosity and the complication of an unwieldy differential equation that cannot be solved analytically make this method unattractive. The remaining method is the theory of structured continua. There are several advantages to this approach.

(a) It yields a velocity function that is relatively easy to evaluate.

(b) There are none of the height dependent parameters to be determined.

(c) This seems to be a logical extension of the continuum hypothesis in the sense that the structures are considered to be continuums within continuums.

To be sure, the theory of structured continua involves difficulties. For one thing, there is no physical description or feeling for what the structures are. Without this description the deformation and voriticity fields cannot be measured and hence only one-third of the theory can be investigated experimentally.

VI. Recommendations for Future Research

It is apparent that the observations used in this study are not detailed enough to adequately test the structured fluid approach. In order to obtain more conclusive results two major problems must be solved first:

(1) better, more detailed flow measurements must be otained;

(2) some formal physical explanation for the "st uctures" must be developed.

With the solutions to the above two problems in hand one could use the differential equations (4, 5, 6) to determine all of the viscosity coefficients that permeate the theory.

At this point, a clearly written orderly summary of the theory of structured fluids would be quite beneficial. The abstract formalizations have reached the point where they must be stated concisely and put in their proper perspective so that the interested researcher will have a jumping off point from which he can wade through the material already in print on the subject.

Acknowledgements

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Table 1.

Type of Canopy	Z_H(cm)	U_H(cm/sec)	K	L	Remarks
Japanese Larch	1040	149	3.0	3.5	Can't distinguish graphically between data and fit.
Japanese Larch	1040	348	3.5	3.0	Can't distinguish graphically between data and fit.
Oats	125	61	4.5	3.0	Can't distinguish graphically between data and fit.
Oats	125	169	3.0	3.5	Slight distinction
Soy Beans	150	98	3.0	3.5	No distinction
Soy Beans	150	246	2.5	3.0	Slight distinction
Corn	300	81	4.5	3.0	Slight distinction
Corn	300	300	3.5	3.0	No distinction
Sunflowers	225	225	3.5	3.0	No distinction

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